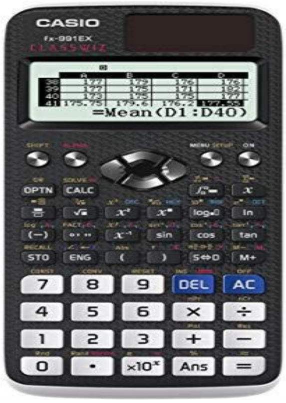
A-Level Maths and Further Maths Summer Bridging Work 2023/24

Exam board: Edexcel

Resources that you need to purchase in preparation for studying this course:

- Calculator: CASIO FX-991EX Advanced Scientific Calculator (UK Version) **Please note:** Your GCSE calculator is not suitable for A-level studies due to the lack of certain functions for statistics and further maths.



- Ring binder/folder
- Dividers for Pure and Applied.

Welcome to A-level Mathematics

In choosing to study mathematics at A-level you have made the choice to study for a qualification which is highly respected throughout the world of education and employment The course is rewarding and demanding and many students find the difference between GCSE and A-level a bit overwhelming at first. This pack is designed to prepare you for the work you will face in the autumn term by helping you to secure the knowledge you gained at GCSE level. Nearly all of the harder algebra you learnt at GCSE appears again at but it is assumed that you will be able to do it without the lesson stopping to go into detail. This pack will also give you a taste of the individual effort and preparation required to be successful at A-level. Studying mathematics is a demanding but ultimately rewarding experience and I want to wish you all good luck in your studies.

Mr B Wilshaw

Learning Area Leader- Mathematics

Topics

1. Factorising
2. Formulae
3. Linear and Quadratic Equations
4. Simultaneous Equations
5. Simplifying including Index Laws Instructions and Deadline

Complete the following sections.

Answer the questions on separate paper showing your working out. Remember to put your name on your answers and you are expected to hand in this pack when you sign up for the course in September. Failure to hand this is in will put you on the 'amber' list of students meaning that you will have to prove your ability to be successful in the first few weeks of the course or be forced to drop the subject.

Topic 1: Factorising

Examples:

1. $2x^3 - 6x^2 = 2x^2(x-3)$ ← Look for the highest common factor in each term.
2. $9a^2 - 4b^2 = (3a+2b)(3a-2b)$ ← Spot that this is a difference of two squares.
3. $x^2 - 7x + 12 = (x-4)(x-3)$ ← Standard factorising of quadratics into 2 brackets
4. $6x^2 + 11x + 4 = (2x+1)(3x+4)$

You can do this last example by trial and error or by the AC method explained below:

$Ax^2 + Bx + C$	All quadratics can be written in this form
$AC = 24$	Multiply A and C (This is where the method gets its name from) Now find 2 numbers which multiply to give AC and add to give B
$6x^2 + 11x + 4$	In this case the numbers are 3x8=24 and 3+8=11. Write the equation
$6x^2 + 3x + 8x + 4$	splitting the Bx term into these two new numbers (here 3 and 8)
$3x(2x+1) + 4(2x+1)$	Now you can factorise the two terms on the left and the two terms on the right separately into 2 brackets.
$(3x+4)(2x+1)$	Finally you should see a common bracketed term and factorise this out.

Exercise:

Factorise completely:

1. $n^2 - np$
2. $h^2 - 25$
3. $m^2 + 7m + 10$
4. $n^2 - n - 12$
5. $15 - 2b - b^2$
6. $3x^2 - 75$
7. $5h^2 - 8h - 4$
8. $10x^2 + 9x + 2$
9. $ab + 5a - 2b - 10$
10. $(x+3)(x+5) + (x+3)^2$

A) Re-arranging Formulae

Examples:

In each case re-arrange the equation to make the letter in brackets the subject of the equation.

1. $y = \frac{x}{2} + 7$ (x) Remember whatever you do to one side do to the other.

   $y - 7 = \frac{x}{2}$

   $2(y - 7) = x$

   $x = 2y - 14$

2. $y = 4(x + 3)$ (x) You don't have to multiply out the brackets.

   $\frac{y}{4} = x + 3$

   $x = \frac{y}{4} - 3$

3. $a^2 = b^2 - 7ac$ (b) Don't forget that when you square root the answers could be + or –.

   $a^2 + 7ac = b^2$

   $b = \pm\sqrt{a^2 + 7ac}$

4. $\frac{3}{m} = 6 - 4n$ (m) Don't overlook the usefulness of brackets in your work.

   $3 = m(6 - 4n)$

   $\frac{3}{(6 - 4n)} = m$

Exercise:

Re-arrange each formula to make the letter in the bracket the subject.

1. $y = \sqrt{3x}$ (x)

2. $4y = 2x - 7$ (x)

3. $3(y + 2) = 6 - 3(x + 7)$ (x)

4. $ab - cd = 4e$ (c)

5. $3a(x + y) = 2b^2$ (a)

6. $\frac{3y}{x} = \frac{x}{4z}$ (x)

7. $bx + cy = d^2$ (c)

8. $k(l - m) = l(m - n)$ (l)

B) Substitution into formulae

Examples:

Evaluate (recall that this means calculate) x if a=0.7, b=-3.5 and c=-2.15. Give answers to 3 s.f.

1.
$$x = ab - c^2$$
$$x = 0.7 \times (-3.5) - (-2.15^2)$$
$$x = -7.07$$

2.
$$x = \sqrt{a(b^2 + c)}$$
$$x = \sqrt{0.7[(-3.5)^2 - 2.15]}$$
$$x = 2.66$$

Exercise:

Use the values a=0.7, b=-3.5 and c=-2.15 to evaluate x in each case. Give your answers to 3 S.f.

1. $x = 4bc + a^2$

2. $x = \dfrac{ac + b^2}{c}$

3. $x = \sqrt{b^2 - \dfrac{c}{a}}$

4. $7x = 2a - 3b + 4c$

5. $ax + bx = c$

Topic 3: Linear and Quadratic Equations

A) Linear Equations

Examples:

Solve:

1.

$$5(x-3)+2=8$$
$$5x-15+2=8$$
$$5x=21$$
$$x=4.2$$

2.

$$\frac{4(x-1)}{7}-\frac{3(1-x)}{4}=2$$

Cross multiply to put over a common denominator.

$$\frac{16(x-1)-21(1-x)}{7\times 4}=2$$
$$\frac{16(x-1)-21(1-x)}{28}=2$$

Clear the fraction by multiplying by 28 and expand and simplify

$$16(x-1)-21(1-x)=56$$
$$16x-16-21+21x=56$$
$$37x-37=56$$
$$37x=93$$
$$x=\frac{93}{37}=2\frac{19}{37}$$

Exercise:

Solve:

1. $3(b+7)=8(2b-3)$

2. $\frac{3}{4}c=\frac{4}{5}$

3. $(3-x)-(3x-3)=30$

4. $\frac{1}{2}(2x+1)+\frac{1}{3}(9x-10)=0$

5. $\frac{2(x-3)}{3}+\frac{4(1-2x)}{5}=1$

6. $7(3x-4)-8=4-2(x-3)$

7. $\frac{a}{2}-\frac{a}{3}=7$

8. $\frac{2(x+5)}{3}=\frac{3(2x-3)}{4}$

B) Quadratic Equations

Examples:

1. Simple factors (remember to find two numbers which multiply to give c and add to give b in $ax^2+bx+c=0$)

$$x^2+7x+12=0$$
$$(x+3)(x+4)=0$$
$$x=-3$$
$$x=-4$$

(using 3x4=12 and 3+4=7)

2. Simple factors using the AC method as outlined in the section on factorising.

$$6x^2+7x-3=0$$
$$6x^2-2x+9x-3=0$$
$$2x(3x-1)+3(3x-1)=0$$
$$(2x+3)(3x-1)=0$$
$$2x+3=0 \qquad 3x-1=0$$
$$x=\frac{-3}{2} \qquad x=\frac{1}{3}$$

(AC=6x-3=-18. Use -2 and 9 as -2+9=7 and -2x9=-18)

(We get two answers as expected for this quadratic)

3. If the quadratic won't factorise we can consider using the quadratic formula which states that, for any quadratic of the form:

$$ax^2+bx+c=0 \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example:

Solve: $3x^2-7x-2=0$

$$x=\frac{7\pm\sqrt{(-7)^2-4\times3\times-2}}{2\times3}$$
$$x=\frac{7\pm\sqrt{49+24}}{6}$$
$$x=\frac{7\pm\sqrt{73}}{6} \qquad x=\frac{7+\sqrt{73}}{6}=2.59 \qquad x=\frac{7-\sqrt{73}}{6}=-0.26$$

Exercise:

Solve:

1. $(x-6)(x+2)=0$
2. $x(x+1)=0$
3. $x^2+5x+6=0$
4. $x^2-7x=-10$
5. $x^2=2x$
6. $2x^2-3x-2=0$
7. $x^2+6x+4=0$
8. $2k^2+4k-3=0$
9. $4p^2+7p=6$
10. $\frac{3}{x+1}=x$

Topic 4: Simultaneous Equations

The trick to simultaneous equations is to make the numbers in front (called the coefficients) of x or y to be equal. Then you can either add or subtract the equations to eliminate one of the letters (variables). You then solve what is left. Once you have the value of one variable don't forget to find the other one by substituting your answer back into any one of the equations.

Examples:

1.

$x + y = 8 \quad (1)$

$4x - y = -3 \quad (2)$

$(1)+(2)$

$$\begin{array}{r} x + y = 8 \\ +\ 4x - y = -3 \\ \hline 5x \quad = 5 \end{array}$$

$x = 1$

$x + y = 8 \quad (1)$

$1 + y = 8$

$y = 7$

> Notice that the coefficients of y in equations (1) and (2) allow you to add the two equations to eliminate y. This allows you to find x. Now substitute this value of x back into an equation, here I have used (1), to find y.

2.

$5x + 3y = 2 \quad (1)$

$2x + y = 0 \quad (2)$

$\times(2)\ by\ 3$

$6x + 3y = 0 \quad (3)$

> In this example none of the coefficients are the same but they can easily be made the same by multiplying one of the equations by 3.

$(3)-(1)$

$$\begin{array}{r} 6x + 3y = 0 \\ -\quad 5x + 3y = 2 \\ \hline x \quad = -2 \end{array}$$

$2x + y = 0 \quad (2)$

$-4 + y = 0$

$y = 4, \quad x = -2$

3.

$8p - 7q = 13 \quad (1)$

$3p + 2q = 28 \quad (2)$

$\times(1)\ by\ 2$

$16p - 14q = 26 \quad (3)$

$\times(2)\ by\ 7$

$21p + 14q = 196 \quad (4)$

> In this example the coefficients can't easily be made equal. You have to multiply both equations.

$(3)+(4)$

$$\begin{array}{r} 16p - 14q = 26 \\ +\ 21p + 14q = 196 \\ \hline 37p \quad = 222 \end{array}$$

$p = \dfrac{222}{37} = 6$

$3p + 2q = 28 \quad (2)$

$3 \times 6 + 2q = 28$

$q = 5, \quad p = 6$

Exercise:

Solve:

1. $$3x - 2y = 10$$
$$x + 2y = 6$$

2. $$4x + 2y = 11$$
$$3x + 4y = 5$$

3. $$2x - 5y = 7$$
$$3x - 4y = 6$$

4. $$y = 3x + 9$$
$$2x + 15 = 3y + 2$$

<u>Topic 5: Simplifying, including index laws</u>

Recall the laws of indices from your GCSE course:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$a^0 = 1$$
$$a^{-n} = \frac{1}{a^n}$$
$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Examples:

1. $a^6 \times a^3 = a^9$

2. $m^5 \div m^3 = m^2$

3. $(x^m)^n = x^{mn}$

4. $$a^3(a^4 + 2ab)$$
$$= a^7 + 2a^4b$$

5. $$\frac{9a^2b}{c} \div \frac{27ab^2}{c^2} = \frac{9a^2b}{c} \times \frac{c^2}{27ab^2} = \frac{ac}{3b}$$

> Don't forget that a on its own means a^1.
>
> Remember the rule for dividing by a fraction. Flip the 2nd fraction and turn it into a multiply.

Exercise:

In both exercises simplify the expression as much as possible using the laws of indices.

	A)		B)
1.	$a^4 \times a^3$	1.	$a^3 \times a^{-4}$
2.	$a^4 \div a^3$	2.	$x^0 \div x^{-4}$
3.	$3b^5 \div b^7$	3.	$18a^2b^2c^2 \times 2(ab)^{-1}$
4.	$a^2(a^3 + a)$	4.	$\frac{1}{2}c^2 \times \frac{3}{4}(pa)^2$
5.	$x^2(y^2 + xy + z)$	5.	$\left(\frac{3}{4}\right)^{-2}$
6.	$(k^3)^2 + \left(\sqrt{k}\right)^4$	6.	$\left(\frac{3}{4}\right)^{-3} \div 6^{-2}$
7.	$\frac{16ab^2}{c^2} \times \frac{ac}{4b^2}$	7.	$a^7 \div a^{-1}$
8.	$\frac{mn^2}{4} \div \frac{2m}{n}$	8.	$\left(\frac{1}{4}\right)^{-\frac{1}{2}}$
9.	$\frac{a^3(a^2 - b^2)}{a^5}$	9.	$15a^2b^7 \div 5ab^{-2}$
10.	$\frac{3x^2(y^2 - x)}{9} \times \frac{x^4}{4}$	10.	$\left(c^2d^4\right)^{\frac{1}{2}} \div c^{-1}$